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**A MULTI-ECHELON SUPPLY CHAIN INVENTORY MODEL WITH QUADRATIC DEMAND RATE AND VARIABLE DETERIORATION**

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**ABSTRACT:** In this paper, an integrated supply chain model has been developed under the assumption of demand rate is quadratic form which starts from zero in the beginning and ends to zero at the completion of the cycle. The variable rate of deterioration has been taken. In this model, three suppliers, one distributor and three retailers are considered. The replenishment of items at the distributor and retailers is considered as instantaneous. Shortage of items is not allowed. Cost minimization technique has been used to obtain the optimal values of the parameter.

**KEYWORDS:** Deterioration, supply chain, three- echelon inventory system.

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**INTRODUCTION**

In recent year the interest of researchers has been increased in a study of supply chain management. A supply chain is a network of suppliers, manufacturers, distributors and retailers that perform the function of procurement of material, transformation of these materials into intermediate and finished product and distribution of this product to customers. This chain is traditionally characterized by forward flow of materials and backward flow of information. The three key members of the supply chain (supplier, distributor and the retailer) managed independently their inventories. Increasing competitive pressure and due to decrease in margin of profit the company force to develop supply chain which provide quick response to customers' needs and reducing the cost of carrying material. The number of deliveries is derived in cooperation with each other to achieve a minimum total integrated cost.

Clark A. J. Scarf Herbert [2] in year 1960 was the first to consider the echelon stock in the inventory system. Bessler S. A. and Veinott A.F. [1] in year 1966 discussed the optimal policy for a dynamic multi-echelon inventory model.

Wee H. M [19] developed a model with an exponential decaying inventory to determine optimal joint total relevant annual cost of the buyer and seller.

He extended the research to an all-unit quantity discount inventory policy for deteriorating items with various deteriorating rates and negotiation factors and determined the optimal replenishment interval and discount price so that the joint total annual cost is minimized.

Risk reduction of items shortage is also an important factor in supply management. The shortage of items may be avoided through suppliers in a supply chain. In earlier literature, it is assumed that inventory is replenished by a single supplier. But in reality there are situations when more than one supplier is required to satisfy a desirable goal. Sedarage Dayani, Fujiwala O. and Luong, Huynh Trang [17] presented mathematical model of N-supplier inventory systems and derived optimal inventory policy. Ranga V. Ramasesh, J. Keith Ord and Jack C. Hayya [15] showed that dual sourcing is required when requirement is large or lead time is uncertain. Hon Shiang Lau and Long Geng Zha [11] presented optimal ordering policies with two suppliers when lead times and demand are all stochastic.

Yu [21] develops a two- supplier three- echelon inventory model that integrates the upper, middle and lower levels in supply chain. He describes how to integrated approach to decision making can achieve global optimum as compared to independent decision by the supplier, the distributor and the retailer. Mishra P.N

and Goyal Manish [13] develop a multi-echelon supply chain inventory model with linear demand rate and variable deterioration. In this paper, we extended our research paper [13] by considering demand rate as of quadratic form which starts from zero and ends with zero. Shortage of items is not allowed. Cost Minimization technique has been used to obtain the optimal values of the parameter.

### ASSUMPTIONS AND NOTATIONS

#### ASSUMPTION

The following assumptions are considered in this paper

- (i) A single –product item is considered.
- (ii) The demand rate is quadratic function of time.
- (iii) No shortage is allowed.
- (iv) The rate of replenishment at the distributor and the retailers is instantaneous.
- (v) There is no constraint in space or capital.
- (vi) The variable rate of deterioration has been taken.

#### NOTATIONS:

The following notations have been used in this paper

- TC : The joint cost per unit time for the supply chain.
- $TC_r$  : The total cost per unit time for the retailers.
- $TC_d$  : The total cost per unit time for the distributor.
- $TC_s$  : The total cost per unit time for the suppliers.
- $I_{rk}(t)$ : Inventory level of retailer  $k$  at time  $t$ .
- $I_d(t)$  : Inventory level of distributor at time  $t$ .
- $I_{si}(t)$  : Inventory level of supplier  $i$  at time  $t$  where  $i=1, 2$
- $C_{rk}$  : The ordering cost for retailer  $k$ .
- $C_d$  : The ordering cost for the distributor.
- $C_{si}$  : The ordering cost for supplier  $i$ .
- $H_{rk}$  : The carrying cost per unit per unit time for retailer  $k$ , where  $k=1, 2, 3$ .
- $H_d$  : The carrying cost unit per unit time for distributor
- $H_{si}$  : The carrying cost per unit per unit time for supplier  $i$ , where  $i=1, 2$ .
- $D_{rk}$  : The deterioration cost per unit per unit time for retailer  $k$  where  $k=1, 2, 3$ .
- $D_d$  : The deterioration cost per unit per unit time for distributor.
- $D_{si}$  : The deterioration cost per unit per unit time for supplier  $i$ .  $i=1, 2$ .
- $\theta_0 t$  : The variable deterioration rate of inventory
- $\mu$ : The life time after which the deterioration of items starts.
- $T$ : The supply cycle time for the producer.
- $t_{rk}$  : The replenishment cycle time for retailer  $k$ .
- $t_d$ : The replenishment cycle time for distributor.
- $n_{sd}$  : The number of deliveries from the suppliers to the distributor during time  $T$
- $n_{dr}$  : The number of deliveries from the distributor to the retailers during time  $T$

### MATHEMATICAL MODELS AND ANALYSIS

The  $N$ -supplier- one-distributor- $N$ -retailers inventory system is illustrated in fig.1, fig.2 and fig.3 .We consider the supply chain system consisting of three suppliers, one distributor and three retailers. Now we develop the cost structures of suppliers, distributor and the retailers as follows:

## (a) Cost Structure of N-retailers:

We consider the number of retailers  $N$  equal to three in the supply chain. Let  $I_{rk}(t)$  be the retailer's inventory level at any time  $t$  and demand rate be quadratic function given by  $a_k t(t_{rk} - t)$ . Let the inventory starts to deteriorate after the life time  $\mu$  at the rate of  $\theta_0 t$ . The inventory system at the retailer is given in figure-1 given below:

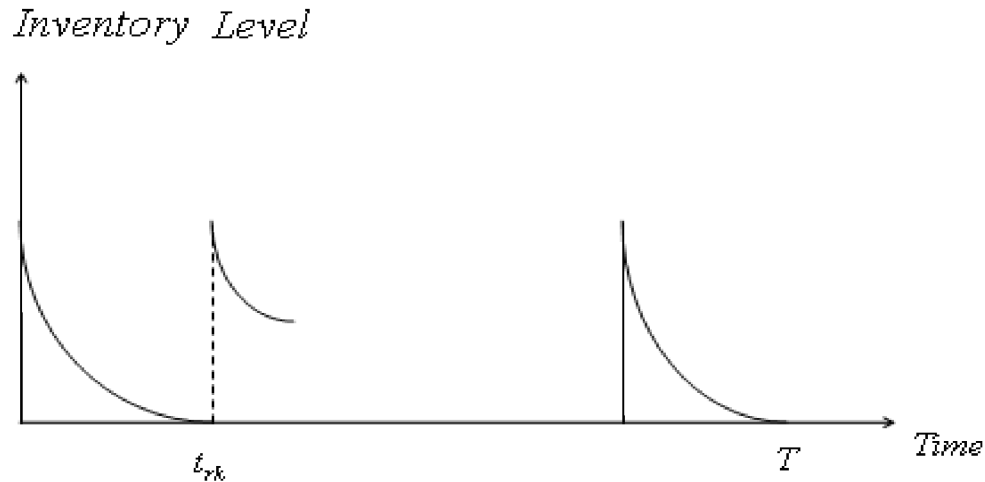


Figure-1

The differential equation governing the retailer's inventory level is given by

$$\frac{dI_{rk}(t)}{dt} = -a_k t(t_{rk} - t), \quad 0 \leq t \leq \mu \quad (1)$$

$$\frac{dI_{rk}(t)}{dt} = -a_k t(t_{rk} - t) - \theta_0 t I_{rk}, \quad \mu \leq t \leq t_{rk} \quad (2)$$

The various boundary conditions are given by

$$I_{rk}(0) = S_{rk}, \quad I_{rk}(t_{rk}) = 0 \quad (3)$$

The solution of equation (1) is given by

$$I_{rk}(t) = -a_k \left( \frac{t_{rk} t^2}{2} - \frac{t^3}{3} \right) + c_1 \quad (4)$$

Using boundary condition (3) in equation (4), we get

$$c_1 = S_{rk}$$

Now substituting the value of  $c_1$  in equation (4), we get

$$I_{rk}(t) = -a_k \left( \frac{t_{rk} t^2}{2} - \frac{t^3}{3} \right) + S_{rk}, \quad 0 \leq t \leq \mu \quad (5)$$

The solution of equation (2) is given by

$$I_{rk}(t) = -a_k \left[ \frac{t_{rk} t^2}{2} - \frac{t^3}{3} + \frac{\theta_0}{2} \left( \frac{t_{rk} t^4}{4} - \frac{t^5}{5} \right) \right] e^{-\frac{\theta_0 t^2}{2}} + c_2 e^{-\frac{\theta_0 t^2}{2}} \quad (6)$$

Using boundary condition (3) in equation (6), we get

$$c_2 = a_k \left( \frac{1}{6} t_{rk}^3 + \frac{\theta_0}{40} t_{rk}^5 \right)$$

Substituting the value of  $c_2$  in equation (6), we get

$$I_{rk}(t) = -a_k \left[ \frac{t_{rk} t^2}{2} - \frac{t^3}{3} + \frac{\theta_0}{2} \left( \frac{t_{rk} t^4}{4} - \frac{t^5}{5} \right) \right] e^{-\frac{\theta_0 t^2}{2}} + a_k \left( \frac{1}{6} t_{rk}^3 + \frac{\theta_0}{40} t_{rk}^5 \right) e^{-\frac{\theta_0 t^2}{2}} \quad (7)$$

Where,  $\mu \leq t \leq t_{rk}$

From (5) and (7), we have

$$-a_k \left( \frac{t_{rk}\mu^2}{2} - \frac{\mu^3}{3} \right) + S_{rk} = -a_k \left[ \frac{t_{rk}\mu^2}{2} - \frac{\mu^3}{3} + \frac{\theta_0}{2} \left( \frac{t_{rk}\mu^4}{4} - \frac{\mu^5}{5} \right) \right] e^{-\frac{\theta_0\mu^2}{2}} + a_k \left( \frac{1}{6} t_{rk}^3 + \frac{\theta_0}{40} t_{rk}^5 \right) e^{-\frac{\theta_0\mu^2}{2}}$$

$$\therefore S_{rk} = a_k \left( \frac{t_{rk}\mu^2}{2} - \frac{\mu^3}{3} \right) - a_k \left[ \frac{t_{rk}\mu^2}{2} - \frac{\mu^3}{3} + \frac{\theta_0}{2} \left( \frac{t_{rk}\mu^4}{4} - \frac{\mu^5}{5} \right) \right] e^{-\frac{\theta_0\mu^2}{2}} + a_k \left( \frac{1}{6} t_{rk}^3 + \frac{\theta_0}{40} t_{rk}^5 \right) e^{-\frac{\theta_0\mu^2}{2}} \quad (8)$$

The deteriorating quantities per replenishment cycle can be obtained as

$$R_{rk} = S_{rk} - \int_0^{t_{rk}} a_k t (t_{rk} - t) dt$$

$$= S_{rk} - \frac{a_k}{6} t_{rk}^3 \quad (9)$$

$$= a_k \left( \frac{t_{rk}\mu^2}{2} - \frac{\mu^3}{3} \right) - a_k \left[ \frac{t_{rk}\mu^2}{2} - \frac{\mu^3}{3} + \frac{\theta_0}{2} \left( \frac{t_{rk}\mu^4}{4} - \frac{\mu^5}{5} \right) \right] e^{-\frac{\theta_0\mu^2}{2}} + a_k \left( \frac{1}{6} t_{rk}^3 + \frac{\theta_0}{40} t_{rk}^5 \right) e^{-\frac{\theta_0\mu^2}{2}} - \frac{a_k}{6} t_{rk}^3 = A_{rk} \theta_0 t \quad (10)$$

The deterioration cost per replenishment cycle is given by

$$DC_{rk} = R_{rk} \times D_{rk}$$

$$= \left[ a_k \left( \frac{t_{rk}\mu^2}{2} - \frac{\mu^3}{3} \right) - a_k \left[ \frac{t_{rk}\mu^2}{2} - \frac{\mu^3}{3} + \frac{\theta_0}{2} \left( \frac{t_{rk}\mu^4}{4} - \frac{\mu^5}{5} \right) \right] e^{-\frac{\theta_0\mu^2}{2}} + a_k \left( \frac{1}{6} t_{rk}^3 + \frac{\theta_0}{40} t_{rk}^5 \right) e^{-\frac{\theta_0\mu^2}{2}} - \frac{a_k}{6} t_{rk}^3 \right] D_{rk} \quad (11)$$

From equation (10), we have

$$A_{rk} = \frac{a_k}{\theta_0 t} \left( \frac{t_{rk}\mu^2}{2} - \frac{\mu^3}{3} \right) - \frac{a_k}{\theta_0 t} \left[ \frac{t_{rk}\mu^2}{2} - \frac{\mu^3}{3} + \frac{\theta_0}{2} \left( \frac{t_{rk}\mu^4}{4} - \frac{\mu^5}{5} \right) \right] e^{-\frac{\theta_0\mu^2}{2}} + \frac{a_k}{\theta_0 t} \left( \frac{1}{6} t_{rk}^3 + \frac{\theta_0}{40} t_{rk}^5 \right) e^{-\frac{\theta_0\mu^2}{2}} - \frac{a_k}{6\theta_0 t} t_{rk}^3$$

The inventory carrying cost per replenishment cycle is given by

$$HC_{rk} = A_{rk} \times H_{rk} = \left[ \frac{a_k}{\theta_0 t} \left( \frac{t_{rk}\mu^2}{2} - \frac{\mu^3}{3} \right) - \frac{a_k}{\theta_0 t} \left[ \frac{t_{rk}\mu^2}{2} - \frac{\mu^3}{3} + \frac{\theta_0}{2} \left( \frac{t_{rk}\mu^4}{4} - \frac{\mu^5}{5} \right) \right] e^{-\frac{\theta_0\mu^2}{2}} + \frac{a_k}{\theta_0 t} \left( \frac{1}{6} t_{rk}^3 + \frac{\theta_0}{40} t_{rk}^5 \right) e^{-\frac{\theta_0\mu^2}{2}} - \frac{a_k}{6\theta_0 t} t_{rk}^3 \right] H_{rk} \quad (12)$$

The retailer's total cost per replenishment cycle can be expressed as the sum of ordering cost transportation cost, the carrying cost and the deterioration cost. Thus we have

$$TC_r(t_{rk}) = \sum_{i=1}^3 [C_{rk} + \left[ \frac{a_k}{\theta_0 t} \left( \frac{t_{rk}\mu^2}{2} - \frac{\mu^3}{3} \right) - \frac{a_k}{\theta_0 t} \left[ \frac{t_{rk}\mu^2}{2} - \frac{\mu^3}{3} + \frac{\theta_0}{2} \left( \frac{t_{rk}\mu^4}{4} - \frac{\mu^5}{5} \right) \right] e^{-\frac{\theta_0\mu^2}{2}} + \frac{a_k}{\theta_0 t} \left( \frac{1}{6} t_{rk}^3 + \frac{\theta_0}{40} t_{rk}^5 \right) e^{-\frac{\theta_0\mu^2}{2}} - \frac{a_k}{6\theta_0 t} t_{rk}^3 \right] H_{rk} + \left[ a_k \left( \frac{t_{rk}\mu^2}{2} - \frac{\mu^3}{3} \right) - a_k \left[ \frac{t_{rk}\mu^2}{2} - \frac{\mu^3}{3} + \frac{\theta_0}{2} \left( \frac{t_{rk}\mu^4}{4} - \frac{\mu^5}{5} \right) \right] e^{-\frac{\theta_0\mu^2}{2}} + a_k \left( \frac{1}{6} t_{rk}^3 + \frac{\theta_0}{40} t_{rk}^5 \right) e^{-\frac{\theta_0\mu^2}{2}} - \frac{a_k}{6} t_{rk}^3 \right] D_{rk} + (a + bQ_r)] \quad (13)$$

The retailer's total cost during cycle T is given by

$$TC_r(t) = (n_{sd} n_{dr}) \sum_{i=1}^3 \left[ C_{rk} + \left[ \frac{a_k}{\theta_0 t} \left( \frac{T}{n_{sd} n_{dr}} \frac{\mu^2}{2} - \frac{\mu^3}{3} \right) - \frac{a_k}{\theta_0 t} \left[ \left( \frac{T}{n_{sd} n_{dr}} \frac{\mu^2}{2} - \frac{\mu^3}{3} \right) + \frac{\theta_0}{2} \left( \frac{T}{n_{sd} n_{dr}} \frac{\mu^4}{4} - \frac{\mu^5}{5} \right) \right] e^{-\frac{\theta_0\mu^2}{2}} - \left( \frac{1}{6} \frac{T^3}{(n_{sd} n_{dr})^3} \frac{\mu^2}{2} + \frac{\theta_0}{40} \frac{T^5}{(n_{sd} n_{dr})^5} \right) e^{-\frac{\theta_0\mu^2}{2}} + \frac{a_k}{6} \frac{T^3}{(n_{sd} n_{dr})^3} \right] H_{rk} + \left[ a_k \left( \frac{T}{n_{sd} n_{dr}} \frac{\mu^2}{2} - \frac{\mu^3}{3} \right) - a_k \left[ \frac{T}{n_{sd} n_{dr}} \frac{\mu^2}{2} - \frac{\mu^3}{3} + \frac{\theta_0}{2} \left( \frac{T}{n_{sd} n_{dr}} \frac{\mu^4}{4} - \frac{\mu^5}{5} \right) \right] e^{-\frac{\theta_0\mu^2}{2}} + a_k \left( \frac{1}{6} \frac{T^3}{(n_{sd} n_{dr})^3} + \frac{\theta_0}{40} \frac{T^5}{(n_{sd} n_{dr})^5} \right) e^{-\frac{\theta_0\mu^2}{2}} - \frac{1}{6} \frac{T^3}{(n_{sd} n_{dr})^3} \right] D_{rk} + (a + bQ_r) \right] \quad (14)$$

## (b) Cost Structure of Distributor

We consider one distributor in the supply chain. Let  $I_d(t)$  be the inventory level at the distributor at any time  $t$ . The demand rate at the distributor is given by  $\sum_{k=1}^3 a_k t(t_{rk} - t)$ . The deterioration rate of the inventory is  $\theta_0 t$ . The inventory system at the distributor is illustrated in figure-2

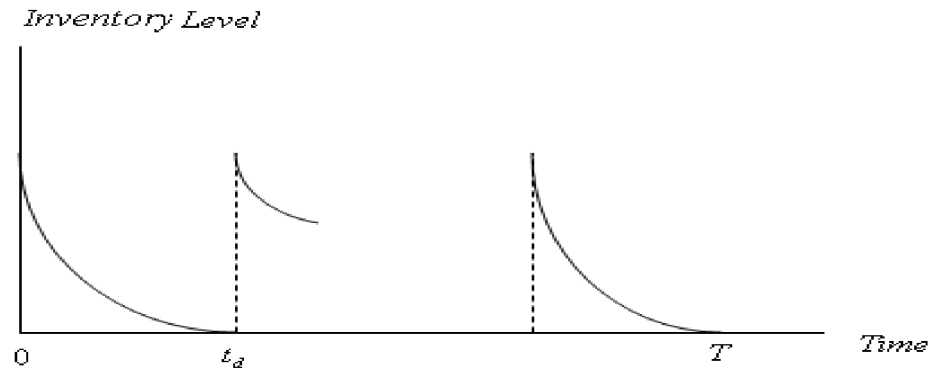


Figure-2

The differential equation governing the inventory at the distributor is given by

$$\frac{dI_d(t)}{dt} = -\sum_{k=1}^3 a_k t(t_d - t), \quad 0 \leq t \leq \mu \quad (15)$$

$$\frac{dI_d(t)}{dt} = -\sum_{k=1}^3 a_k t(t_d - t) - \theta_0 t I_d, \quad \mu \leq t \leq t_d \quad (16)$$

The various boundary conditions are given by

$$I_d(0) = S_d, \quad I_d(t_d) = 0 \quad (17)$$

The solution of equation (15) is given by

$$I_d(t) = -\sum_{k=1}^3 a_k \left( \frac{t_d t^2}{2} - \frac{t^3}{3} \right) + c_3 \quad (18)$$

Using boundary condition (17) in equation (18), we get

$$c_3 = S_d$$

Now substituting the value of  $c_3$  in equation (18), we get

$$I_d(t) = -\sum_{k=1}^3 a_k \left( \frac{t_d t^2}{2} - \frac{t^3}{3} \right) + S_d, \quad 0 \leq t \leq \mu \quad (19)$$

The solution of equation (16) is given by

$$I_d(t) = -\sum_{k=1}^3 a_k \left[ \frac{t_d t^2}{2} - \frac{t^3}{3} + \frac{\theta_0}{2} \left( \frac{t_d t^4}{4} - \frac{t^5}{5} \right) \right] e^{-\frac{\theta_0 t^2}{2}} + c_4 e^{-\frac{\theta_0 t^2}{2}} \quad (20)$$

Using boundary condition (17) in equation (20), we get

$$c_4 = \sum_{k=1}^3 a_k \left( \frac{1}{6} t_d^3 + \frac{\theta_0}{40} t_d^5 \right)$$

Substituting the value of  $c_4$  in equation (20), we get

$$I_d(t) = -\sum_{k=1}^3 a_k \left[ \frac{t_d t^2}{2} - \frac{t^3}{3} + \frac{\theta_0}{2} \left( \frac{t_d t^4}{4} - \frac{t^5}{5} \right) \right] e^{-\frac{\theta_0 t^2}{2}} + \sum_{k=1}^3 a_k \left( \frac{1}{6} t_d^3 + \frac{\theta_0}{40} t_d^5 \right) e^{-\frac{\theta_0 t^2}{2}} \quad (21)$$

Where,  $\mu \leq t \leq t_d$

From (19) and (21), we have

$$\begin{aligned}
 & - \sum_{k=1}^3 a_k \left( \frac{t_d \mu^2}{2} - \frac{\mu^3}{3} \right) + S_d = \\
 & - \sum_{k=1}^3 a_k \left[ \frac{t_d \mu^2}{2} - \frac{\mu^3}{3} + \frac{\theta_0}{2} \left( \frac{t_d \mu^4}{4} - \frac{\mu^5}{5} \right) \right] e^{-\frac{\theta_0 \mu^2}{2}} + \sum_{k=1}^3 a_k \left( \frac{1}{6} t_d^3 + \frac{\theta_0}{40} t_d^5 \right) e^{-\frac{\theta_0 \mu^2}{2}} \\
 \therefore S_d &= \sum_{k=1}^3 a_k \left( \frac{t_d \mu^2}{2} - \frac{\mu^3}{3} \right) \\
 & - \sum_{k=1}^3 a_k \left[ \frac{t_d \mu^2}{2} - \frac{\mu^3}{3} + \frac{\theta_0}{2} \left( \frac{t_d \mu^4}{4} - \frac{\mu^5}{5} \right) \right] e^{-\frac{\theta_0 \mu^2}{2}} + \sum_{k=1}^3 a_k \left( \frac{1}{6} t_d^3 + \frac{\theta_0}{40} t_d^5 \right) e^{-\frac{\theta_0 \mu^2}{2}}
 \end{aligned} \tag{22}$$

The deteriorating quantities per replenishment cycle can be obtained as

$$\begin{aligned}
 R_d &= S_d - \sum_{k=1}^3 \int_0^{t_d} a_k t(t_d - t) dt \\
 &= S_d - \sum_{k=1}^3 \frac{a_k}{6} t_d^3 \\
 &= \sum_{k=1}^3 a_k \left( \frac{t_d \mu^2}{2} - \frac{\mu^3}{3} \right) \\
 & - \sum_{k=1}^3 a_k \left[ \frac{t_d \mu^2}{2} - \frac{\mu^3}{3} + \frac{\theta_0}{2} \left( \frac{t_d \mu^4}{4} - \frac{\mu^5}{5} \right) \right] e^{-\frac{\theta_0 \mu^2}{2}} + \sum_{k=1}^3 a_k \left( \frac{1}{6} t_d^3 + \frac{\theta_0}{40} t_d^5 \right) e^{-\frac{\theta_0 \mu^2}{2}} \\
 & - \sum_{k=1}^3 \frac{a_k}{6} t_d^3 = A_d \theta_0 t
 \end{aligned} \tag{23}$$

The deterioration cost per replenishment cycle is given by

$$\begin{aligned}
 DC_d &= R_d \times D_d \\
 &= \left[ \sum_{k=1}^3 a_k \left( \frac{t_d \mu^2}{2} - \frac{\mu^3}{3} \right) - \sum_{k=1}^3 a_k \left[ \frac{t_d \mu^2}{2} - \frac{\mu^3}{3} + \frac{\theta_0}{2} \left( \frac{t_d \mu^4}{4} - \frac{\mu^5}{5} \right) \right] e^{-\frac{\theta_0 \mu^2}{2}} \right. \\
 & \quad \left. + \sum_{k=1}^3 a_k \left( \frac{1}{6} t_d^3 + \frac{\theta_0}{40} t_d^5 \right) e^{-\frac{\theta_0 \mu^2}{2}} - \sum_{k=1}^3 \frac{a_k}{6} t_d^3 \right] D_d
 \end{aligned} \tag{24}$$

From equation (23), we have

$$\begin{aligned}
 A_d &= \frac{1}{\theta_0 t} \left[ \sum_{k=1}^3 a_k \left( \frac{t_d \mu^2}{2} - \frac{\mu^3}{3} \right) - \sum_{k=1}^3 a_k \left[ \frac{t_d \mu^2}{2} - \frac{\mu^3}{3} + \frac{\theta_0}{2} \left( \frac{t_d \mu^4}{4} - \frac{\mu^5}{5} \right) \right] e^{-\frac{\theta_0 \mu^2}{2}} + \sum_{k=1}^3 a_k \left( \frac{1}{6} t_d^3 + \right. \right. \\
 & \quad \left. \left. \frac{\theta_0}{40} t_d^5 \right) e^{-\frac{\theta_0 \mu^2}{2}} - \sum_{k=1}^3 a_k \frac{1}{6} t_d^3 \right]
 \end{aligned} \tag{25}$$

The inventory carrying cost per replenishment cycle is given by

$$\begin{aligned}
 HC_d &= A_d \times H_d \\
 &= \frac{H_d}{\theta_0 t} \left[ \sum_{k=1}^3 a_k \left( \frac{t_d \mu^2}{2} - \frac{\mu^3}{3} \right) - \sum_{k=1}^3 a_k \left[ \frac{t_d \mu^2}{2} - \frac{\mu^3}{3} + \frac{\theta_0}{2} \left( \frac{t_d \mu^4}{4} - \frac{\mu^5}{5} \right) \right] e^{-\frac{\theta_0 \mu^2}{2}} + \right. \\
 & \quad \left. \sum_{k=1}^3 a_k \left( \frac{1}{6} t_d^3 + \frac{\theta_0}{40} t_d^5 \right) e^{-\frac{\theta_0 \mu^2}{2}} - \sum_{k=1}^3 a_k \frac{1}{6} t_d^3 \right]
 \end{aligned} \tag{26}$$

By summing the ordering cost, purchase cost, the carrying cost and deteriorating cost, the transportation cost, the distributor's total cost during time T unit can be expressed as

$$TC_d(T) = C_d n_{sd} + HC_d n_{sd} + DC_d n_{sd} + (a + bQ_d) \tag{27}$$

### (c) Cost structure of N-suppliers

We consider the number of suppliers N equal to three in the supply chain. Let  $I_{si}(t)$  be the inventory level at the suppliers at any time t. The demand rate at the supplier is give by  $\sum_{k=1}^3 m_i(a_k t + b_k)$ , where  $m_i$

is the proportion of order arrangement for supplier  $i$ ,  $i=1, 2$ . The deterioration rate of the inventory is  $\theta_0 t$ . The inventory system is illustrated in figure-3 given below

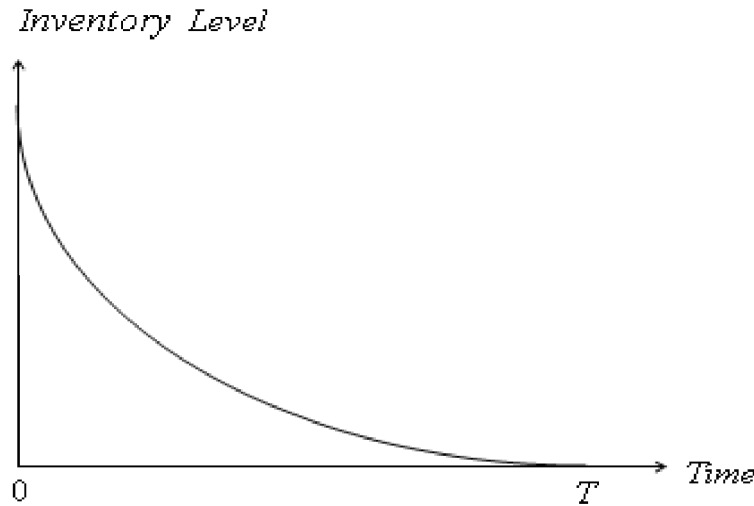


Figure-3

The differential equation governing the inventory at the suppliers is given

$$\frac{dI_{si}(t)}{dt} = - \sum_{k=1}^3 m_i a_k t(T-t) \quad , \quad 0 \leq t \leq \mu \quad (28)$$

$$\frac{dI_{si}(t)}{dt} = - \sum_{k=1}^3 m_i a_k t(T-t) - \theta_0 t I_{si} \quad , \quad \mu \leq t \leq T \quad (29)$$

The various boundary conditions are given by

$$I_{si}(0) = S_{si} \quad , \quad I_{si}(T) = 0 \quad (30)$$

The solution of equation (28) is given by

$$I_{si}(t) = - \sum_{k=1}^3 m_i a_k \left( \frac{Tt^2}{2} - \frac{t^3}{3} \right) + c_4 \quad (31)$$

Using boundary condition (30) in equation (32), we get

$$c_4 = S_{si}$$

Now substituting the value of  $c_4$  in equation (31), we get

$$I_{si}(t) = - \sum_{k=1}^3 m_i a_k \left( \frac{Tt^2}{2} - \frac{t^3}{3} \right) + S_{si} \quad , \quad 0 \leq t \leq \mu \quad (32)$$

The solution of equation (29) is given by

$$I_{si}(t) = - \sum_{k=1}^3 m_i a_k \left[ \frac{Tt^2}{2} - \frac{t^3}{3} + \frac{\theta_0}{2} \left( \frac{Tt^4}{4} - \frac{t^5}{5} \right) \right] e^{-\frac{\theta_0 t^2}{2}} + c_5 e^{-\frac{\theta_0 t^2}{2}} \quad (33)$$

Using boundary condition (30) in equation (33), we get

$$c_5 = \sum_{k=1}^3 m_i a_k \left( \frac{1}{6} T^3 + \frac{\theta_0}{40} T^5 \right)$$

Substituting the value of  $c_5$  in equation (33), we get

$$I_{si}(t) = - \sum_{k=1}^3 m_i a_k \left[ \frac{Tt^2}{2} - \frac{t^3}{3} + \frac{\theta_0}{2} \left( \frac{Tt^4}{4} - \frac{t^5}{5} \right) \right] e^{-\frac{\theta_0 t^2}{2}} + \sum_{k=1}^3 m_i a_k \left( \frac{1}{6} T^3 + \frac{\theta_0}{40} T^5 \right) e^{-\frac{\theta_0 t^2}{2}} \quad (34)$$

Where,  $\mu \leq t \leq T$

From (32) and (34), we have

$$\begin{aligned}
 & -\sum_{k=1}^3 m_i a_k \left( \frac{T\mu^2}{2} - \frac{\mu^3}{3} \right) + S_{si} = \\
 & -\sum_{k=1}^3 m_i a_k \left[ \frac{T\mu^2}{2} - \frac{\mu^3}{3} + \frac{\theta_0}{2} \left( \frac{T\mu^4}{4} - \frac{\mu^5}{5} \right) \right] e^{-\frac{\theta_0 \mu^2}{2}} + \sum_{k=1}^3 m_i a_k \left( \frac{1}{6} T^3 + \frac{\theta_0}{40} T^5 \right) e^{-\frac{\theta_0 \mu^2}{2}} \\
 \\
 & \therefore S_{si} = \sum_{k=1}^3 m_i a_k \left( \frac{T\mu^2}{2} - \frac{\mu^3}{3} \right) \\
 & -\sum_{k=1}^3 m_i a_k \left[ \frac{T\mu^2}{2} - \frac{\mu^3}{3} + \frac{\theta_0}{2} \left( \frac{T\mu^4}{4} - \frac{\mu^5}{5} \right) \right] e^{-\frac{\theta_0 \mu^2}{2}} + \sum_{k=1}^3 m_i a_k \left( \frac{1}{6} T^3 + \frac{\theta_0}{40} T^5 \right) e^{-\frac{\theta_0 \mu^2}{2}} \quad (35)
 \end{aligned}$$

The actual deteriorating quantities at supplier i during supply cycle is given by

$$\begin{aligned}
 R_{si} &= S_{si} - \sum_{k=1}^3 \int_0^T m_i a_k t (T - t) dt - \sum_{k=1}^3 m_i R_d \\
 &= S_{si} - \sum_{k=1}^3 m_i a_k \left[ \frac{1}{6} T^3 \right] - \sum_{k=1}^3 m_i R_d \\
 &= \sum_{k=1}^3 m_i a_k \left( \frac{T\mu^2}{2} - \frac{\mu^3}{3} \right) \\
 & -\sum_{k=1}^3 m_i a_k \left[ \frac{T\mu^2}{2} - \frac{\mu^3}{3} + \frac{\theta_0}{2} \left( \frac{T\mu^4}{4} - \frac{\mu^5}{5} \right) \right] e^{-\frac{\theta_0 \mu^2}{2}} + \sum_{k=1}^3 m_i a_k \left( \frac{1}{6} T^3 + \frac{\theta_0}{40} T^5 \right) e^{-\frac{\theta_0 \mu^2}{2}} \\
 & -\sum_{k=1}^3 m_i a_k \left[ \frac{1}{6} T^3 \right] - \sum_{k=1}^3 m_i R_d = A_{si} \theta_0 t \quad (36)
 \end{aligned}$$

Therefore the deterioration cost at supplier during supply cycle is given by

$$\begin{aligned}
 DC_{si} &= R_{si} \times D_{si} \\
 &= \left[ \sum_{k=1}^3 m_i a_k \left( \frac{T\mu^2}{2} - \frac{\mu^3}{3} \right) \right. \\
 & -\sum_{k=1}^3 m_i a_k \left[ \frac{T\mu^2}{2} - \frac{\mu^3}{3} + \frac{\theta_0}{2} \left( \frac{T\mu^4}{4} - \frac{\mu^5}{5} \right) \right] e^{-\frac{\theta_0 \mu^2}{2}} + \sum_{k=1}^3 m_i a_k \left( \frac{1}{6} T^3 + \frac{\theta_0}{40} T^5 \right) e^{-\frac{\theta_0 \mu^2}{2}} \\
 & \left. -\sum_{k=1}^3 m_i a_k \left[ \frac{1}{6} T^3 \right] - \sum_{k=1}^3 m_i R_d \right] D_{si} \quad (37)
 \end{aligned}$$

From equation (36) and (37) the inventory cost at the suppliers during the cycle can be obtained as

$$HC_{si} = \sum_{i=1}^2 \sum_{k=1}^3 \frac{1}{\theta_0 t} \left[ S_{si} - \sum_{k=1}^3 m_i a_k \frac{T^3}{6} - \sum_{k=1}^3 m_i R_d \right] H_{si}$$

By summing the ordering cost, the purchasing cost, the carrying cost, the retailer's cost total cost per replenishment cycle can be expressed as

$$\begin{aligned}
 TC_{si}(T) &= \sum_{i=1}^2 (C_{si} + HC_{si} + D_{si}) n_{sd} \\
 &= \sum_{i=1}^2 C_{si} n_{sd} + \sum_{i=1}^2 \sum_{k=1}^3 \frac{1}{\theta_0 t} \left[ S_{si} - \sum_{k=1}^3 m_i a_k \frac{T^3}{6} - \sum_{k=1}^3 m_i R_d \right] H_{si} n_{sd} \\
 &+ \left[ S_{si} - \sum_{k=1}^3 m_i a_k \frac{T^3}{6} - \sum_{k=1}^3 m_i R_d \right] D_{si} n_{sd} \quad (38)
 \end{aligned}$$

## **(d) Joint Total Cost Structure of N-suppliers-One-distributor-N-retailer**

Consider three suppliers, one distributor and three retailers in this model. So there are three players (suppliers, distributor and retailers) in the serial inventory of multi-echelon supply chain. The integrated total cost can be presented as

$$TC = TC_r + TC_d + TC_s$$

The optimization problem is a constrained nonlinear program given by

Minimize  $TC = TC_r + TC_d + TC_s$ , subject to condition  $t_{rk} = \frac{T}{n_{sd}n_{dr}}$ ,  $n_{ik} \in N$ ,

Where,  $k=1, 2, 3$  and  $i=1, 2$

The variables of the integrated total cost are  $T$  and  $n_{ik}$ ,  $i=1, 2$  and  $k=1, 2, 3$ .

For the minimization of the integrated total cost  $TC$ , these variables must satisfy the following conditions

$$\frac{dTC}{dT} = 0 \quad (39)$$

$$\text{And } TC(n_{ik}^* - 1) \geq TC(n_{ik}^*) \geq TC(n_{ik}^* + 1) \quad (40)$$

Where  $k=1, 2, 3$  and  $i=1, 2$ ,

The optimal values of  $T$  and  $n_{ik}$  are denoted by  $T^*$  and  $n_{ik}^*$  respectively and these values have to satisfy equations (39) and (40). Also  $n_{ik}^*$  is an integer in vicinity of  $n_{ik}^\#$  that satisfy the condition

$$\frac{dTC}{dn_{ik}^\#} = 0, \text{ where } k=1, 2, 3 \text{ and } i=1, 2$$

## CONCLUSIONS

In this paper, an integrated supply chain model has been developed under the assumption of demand rate is quadratic function of time starting from zero and ending with zero. The variable deterioration rate has been taken. The replenishment of items at the distributor and the retailer is considered as instantaneous. Shortage of the items is not allowed. Cost minimization technique has been used to obtain the optimal values of parameter. This work can further be extended for other form of demand rate, deterioration rate and for the case of shortage.

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